18.06 Professor Edelman Quiz 3 May 4, 2018

Your PRINTED name is: ____________

Please circle your recitation:

(1)	T 10	26-328	D. Kubrak
(2)	T 11	26-328	D. Kubrak
(3)	T 12	4-159	P.B. Alvarez
(7)	T 12	4-153	E. Belmont
(4)	T 1	4-149	P.B. Alvarez
(5)	T 2	4-149	E. Belmont
(6)	T 3	4-261	J. Wang

Note: We are not planning to use gradescope for this exam.

Grading	
1	
2	
3	
Total:	

Your Initials: ____________

1 (30 pts.) Consider the matrices,

$$A(t) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + t \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}.$$

a. (5 pts) Is it possible to find a vector v and a scalar λ that does not depend on t that serves as an eigenvector/eigenvalue for $A(t)$ for all t?

b. (5 pts.) Find both an eigenvector and an eigenvalue of $A(t)$ that does depend on t.

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c. (5 pts.) For which t, if any, is the matrix $A(t)$ not diagonalizable. Explain briefly.

d. (5 pts.) Consider the sequence $x_0 = 0$, $x_1 = 1$, $x_{k+2} = t * x_{k+1} + (1 - t) * x_k$. You can assume $0 < t < 2$. Why does x_k converge to a finite number as $k \to \infty$? Explain briefly.

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e. (10 pts.) (Recommended to do this after completing all other work on the exam.) Calculate the limit of x_k from part d as k goes to infinity. (Hint: Consider the vector $\begin{pmatrix} x_{k+1} \\ x_k \end{pmatrix}$.) (Check: If $t = 1/2$ the limit is $2/3$.)

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2 (30 pts.) **In all cases find a two by two matrix which has the given eigenvalues and the given singular values or explain why it is impossible. Do not use** A^TA **or** AA^T **in any of your explanations.**

a. (5 pts) $\lambda = 0, 1$, $\sigma = 1, 1$

b. (5 pts) $\lambda = 0, 1$ $\sigma = 0, \sqrt{2}$

c. (5 pts) $\lambda = 0, 0$ $\sigma = 0, 2018$

d. (5 pts) $\lambda = i, -i$ $\sigma = 1, 1$

e. (5 pts) $\lambda = 4, 4$ $\sigma = 3, 5$

f. (5 pts.) $\lambda = -1, 1$ $\sigma = \sqrt{(3 \pm \sqrt{5})/2}$ **(You can trust that** $\sigma_1\sigma_2 = 1$ **and** $\sigma_1^2 + \sigma_2^2 = 3$**)**

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3 (40 pts.) **Are the following matrices necessarily positive definite? Explain why or why not?**

a. (5 pts) $A = Q\Lambda Q^T$ **where** Q **is some 4x4 orthogonal matrix and** Λ **is diagonal with** $(1, 2, 3, 4)$ **on the diagonal.**

b. (10 pts) $A = Q_1\Lambda Q_1^T + Q_2\Lambda Q_2^T$**, where** Q_1 **and** Q_2 **are some 4x4 orthogonal matrices and** Λ **is diagonal with** $(1, 2, 3, 4)$ **on the diagonal.**

c. (5 pts) $A = X\Lambda X^T$ **for some matrix** X **and** Λ **is as above? (Hint: Be careful.)**

d. (5 pts.) P **the projection matrix onto** $(1, 2, 3, 4)$**.**

e. (15 pts.) A **is the** n **by** n **tridiagonal matrix with** 2 **for each diagonal entry, and** 1 **for each superdiagonal and subdiagonal entry.** $n = 1, 2, 3, \ldots$**. (Hint: Probably the easiest argument involves computing the determinant of** $T(n)$ **for** $n = 1, 2, 3, \ldots$**.)**

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